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Analogy between turbulent flows in curved pipes and orthogonally rotating pipes

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A quantitative analogy between fully developed turbulent flows in curved pipes and orthogonally rotating pipes will be described through similarity arguments, the use of experimental data and computational results. A pair of similarity parameters will be derived for each turbulent flow, so that they have the same dynamical meaning as those of laminar flows. When the second parameter for each flow is large enough, it will be shown that friction factors, as well as heat transfer rates, of the two flows coincide for equal values of the fundamental parameters. Computed contours of velocity and temperature will also reveal strong similarities between the two flows.

1. Introduction

The secondary flow caused by the Coriolis force in a straight pipe rotating about an axis perpendicular to the pipe axis is similar to that caused by a centrifugal force in a stationary curved pipe. A quantitative analogy between these two flows was drawn for fully developed laminar flows in a previous paper (Ishigaki 1994). It will be shown in this paper that the analogy between these two pipe flows is also valid for turbulent flow.

These two turbulent pipe flows have been investigated separately. Fully developed turbulent flows in curved pipes have been studied experimentally by White (1929), Ito (1959), Seban & McLaughlin (1963), Roger & Mayhew (1964), theoretically by Mori & Nakayama (1967), and computationally by Patankar, Pratap & Spalding (1975). There also have been many studies on turbulent flows in curved ducts of non-circular cross-sections and on developing turbulent flows (see reviews by Nandakumar & Masliyah 1986 and Ito 1987).

Fully developed turbulent flows in orthogonally rotating ducts have been studied experimentally by Trefethen (1957), Ito & Nanbu (1971), theoretically by Mori, Fukada & Nakayama (1971), and computationally by Iacovides & Launder (1991) for rectangular duct flow. A recent survey of the literature on turbulent flows in orthogonally rotating pipes is given in Tehriwal (1994).

It has sometimes been said in qualitative terms that there is an analogy between turbulent flows in stationary curved pipes and in orthogonally rotating straight pipes. In a discussion on Ito's paper, Trefethen (1959) noted that the friction factors for curved pipe flow presented by Ito were approximately similar to those for orthogonally rotating pipe flow, which had measured previously (Trefethen 1957), but he gave no theoretical explanation. Another analogy, between the turbulent parameters describing the effect of buoyancy and describing the effect of streamline curvature or rotation on a turbulent lengthscale, such as mixing length, is also well-known (Bradshaw 1969). This analogy, however, focused only on the turbulent structure in the general flow situation, including external flows, and it did not consider the secondary flows that arise directly from the body force in mean flows. Nor did the authors who studied both of these flows (Ito 1959; Ito & Nanbu 1971 and Mori & Nakayama 1967; Mori *et al.* 1971, and others) mention any relationships between them. Trefethen's discussion on Ito's paper seems to be the only one that directs explicit attention to the analogy between two turbulent duct flows with secondary flows.

Each of the turbulent flows under consideration is generally governed by two parameters. In order to show an analogy between two different flows, it is essential to use proper similarity parameters (dimensionless numbers). It is also essential, as described in the previous paper (Ishigaki 1994), to have a situation where only one of the two parameters governs the flow characteristics in each flow. In laminar flows through curved pipes, the pair of similarity parameters are the Dean number $K_{LC} = Re/\lambda^{1/2}$ and the curvature ratio $\lambda = R/d$, where $Re = w_m d/\nu$ is the Reynolds number in which w_m is the mean velocity of flow, d the diameter of the pipe, ν the fluid kinematic viscosity, and R is the mean radius of curvature of the pipe. The parameters in laminar rotating pipe flow are $K_{LR} = Re/Ro^{1/2}$ and the Rossby number $Ro = w_m/\Omega d$, where Ω is the angular velocity of rotation. K_{LR} and Ro in rotating pipe flow correspond dynamically to K_{LC} and λ in curved pipe flow respectively. These sets have the important property that flow characteristics become independent of λ or Ro when λ or Ro is large enough ('asymptotic invariance property' of the second parameter). Then K_{LC} or K_{LR} is the sole fundamental parameter that determines the dynamical behaviour of each flow. Only then is there an analogy between these laminar flows (Ishigaki 1994).

The purpose of this paper is to introduce the similarity parameters for each turbulent flow corresponding to those for laminar flow, and to reveal the analogy between them; and these are found to be $K_{TC} = Re^{1/4}/\lambda^{1/2}$ and $K_{TR} = Re^{1/4}/Ro^{1/2}$. The second parameters will be the same as those for laminar flows, i.e. λ and Ro. When λ and Ro are large enough, K_{TC} and K_{TR} are the sole parameters. Experimental data will show that the friction factors and heat transfer rates for the two flows are the same when $K_{TC} = K_{TR}$. Computed contours of velocity and temperature will also reveal the similarity between the two flows.

2. Similarity parameters

On reviewing the literature on the turbulent flows under consideration, including flows in rectangular ducts and developing flows, it is found that most authors used the Reynolds number Re to characterize the flows. For example, Patankar *et al.* (1975) used Re and λ to represent the computational results for curved pipe flow. Iacovides & Launder (1991) stated, without any explanation, that the two parameters characterizing the turbulent flow in a rotating duct were Re and $1/Ro$. The Reynolds number, however, may not be a similarity parameter for these flows, since it includes neither the effect of curvature nor the effect of rotation.

Ito (1959) used a dimensionless parameter $Re(d/2R)^2$ to propose a friction formula in curved pipes, and Ito & Nanbu (1971) used $K_t = R_\Omega^2/Re$ and R_Ω to correlate the experimental data on rotating pipe flow, where $R_\Omega = \Omega d^2/\nu$ is the rotational Reynolds number. In both papers, these parameters were derived by an integral method of boundary layer analysis, but neither the physical meaning of the parameters nor the connection between them was mentioned.

In the laminar flow of Ishigaki (1994), governing parameters were derived by applying an appropriate scaling of variables to the Navier–Stokes equations. In turbulent flows, however, the same approach cannot be taken, because unknown turbulent stresses appear in the Reynolds-averaged Navier–Stokes equations. We shall derive the turbulent parameters so that they have the same dynamical meaning as those of laminar flows.

For laminar flow in curved pipes, the Dean number K_{LC} is the square root of the product of (inertia force/viscous force) and (centrifugal force/viscous force). The curvature ratio λ is equal to (inertia force/centrifugal force). A pair of laminar parameters, K_{LR} and *Ro*, in orthogonally rotating pipe flow is obtained by replacing the centrifugal force in K_{LC} and λ with the Coriolis force.

We look for the parameters that have the same dynamical meaning as those for laminar flows. The inertia force $F_i \sim \rho w_m^2/d$ and the centrifugal force $F_c \sim \rho w_m^2/R$ are the same as in laminar flows; only the viscous force differs. In turbulent flow, viscous forces are restricted to an inner wall layer, where the total stress is approximately constant for a moderate pressure gradient and equal to the wall shear stress τ_w. As τ_w can be estimated approximately by using the Blasius formula or, equivalently, the one-seventh law of the velocity profile (Schlichting 1979), the viscous force F_v can be estimated to be $F_v \sim \tau_w/d \sim \rho w_m^{7/4} \nu^{1/4} d^{-5/4}$ as a first approximation. Then we get $K_{TC} = (F_i F_c)^{1/2}/F_v = Re^{1/4}/\lambda^{1/2}$ as a turbulent parameter in curved pipe flow. If F_c is replaced with the Coriolis force $F_r \sim \rho \Omega w_m$, we have $K_{TR} = Re^{1/4}/Ro^{1/2}$ as a turbulent parameter in rotating pipe flow.

The second parameter of each laminar flow, λ and *Ro*, is also valid for the corresponding turbulent flow, since the inertia force and body force have the same forms as in laminar flows. Therefore, analogous to laminar flows, K_{TC} or K_{TR} is the sole governing parameter in each flow when λ or *Ro* is large enough. The analogy between turbulent flows in curved pipes and rotating pipes can only be expected in this case.

Ito's parameter for curved pipe flow corresponds to ${K_{TC}}^4$, on replacing the radius with the diameter of the pipe. The parameter K_t of Ito & Nanbu for rotating pipe flow corresponds with ${K_{TR}}^4$, but the combination of K_t and R_Ω does not have the 'asymptotic invariance property'.

It is useful to comment here that the parameters introduced here may also be valid for developing turbulent flows, if we make use of appropriate dimensionless axial distances. In a study of developing laminar flows, Ishigaki (1993) introduced dimensionless axial distances $Z_C = z/d\lambda^{1/2}$ for curved pipe flow and $Z_R = z/dRo^{1/2}$ for rotating pipe flow. As these distances include no viscous terms, they may also be valid for turbulent flows. Therefore, together with Z_C or Z_R, pairs of parameters introduced here also characterize turbulent developing flows.

3. Verification and discussion

The similarity in each flow and of the analogy between two flows are verified by using available experimental data, supplemented by computational results obtained with a standard $k-\epsilon$ turbulence model because of a lack of experimental data on flow patterns. An outline of the computations is given in the Appendix. For practical purposes, analogy formulae common to both flows are given for the fiction factor and for the heat transfer rate.

3.1. *Friction factor*

Experimental results for the friction factors of the two flows are taken from Ito (1959) for curved pipe flow and from Ito & Nanbu (1971) for rotating pipe flow, as their data are the most comprehensive and precise. The data were obtained for various values of λ or *Ro* larger than about 8. These results, modified by using K_{TC} and K_{TR} respectively, are shown in figure 1, together with an empirical formula by Blasius for non-rotating

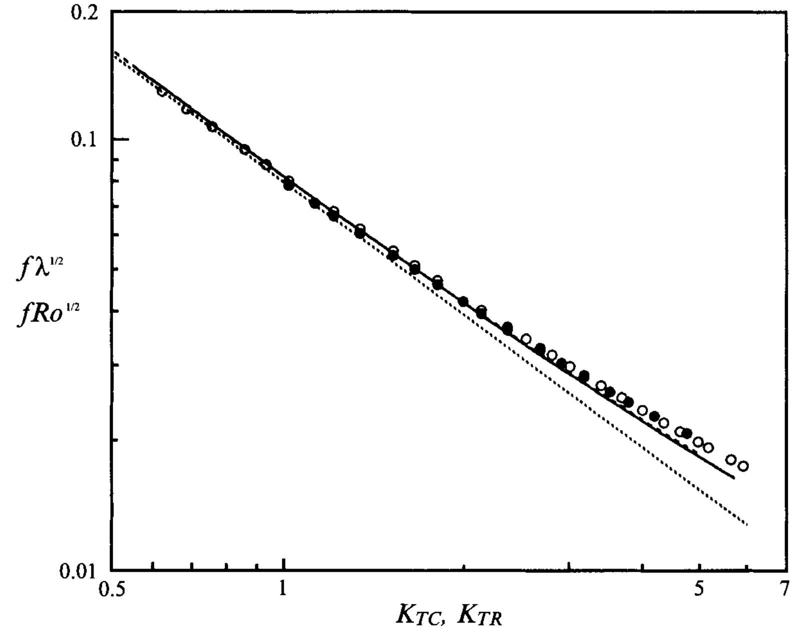

FIGURE 1. Friction factors. Present computation (——, curved pipe flow; -----, rotating pipe flow); ....., Blasius formula for non-rotating straight pipe flow; ○, experimental formula from Ito (1959) (curved flow); ●, experimental formula from Ito & Nanbu (1971) (rotating pipe flow).

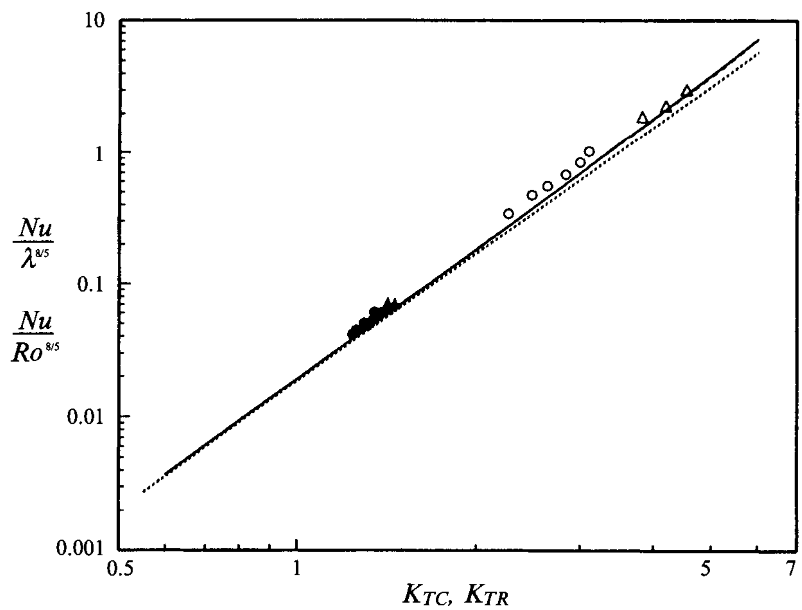

FIGURE 2. Mean Nusselt numbers for a Prandtl number of 0.71. Present computations (——, curved pipe flow; -----, rotating pipe flow); ....., Kays & Crawford (1980) formula for non-rotating straight pipe flow; ○, △, experimental data from Mori & Nakayama (1967) for curved pipe flow ($\lambda = 9.35$ and 20); ●, ▲, experimental data from Mori *et al.* (1971) for rotating pipe flow ($Ro = 10$ and 100).

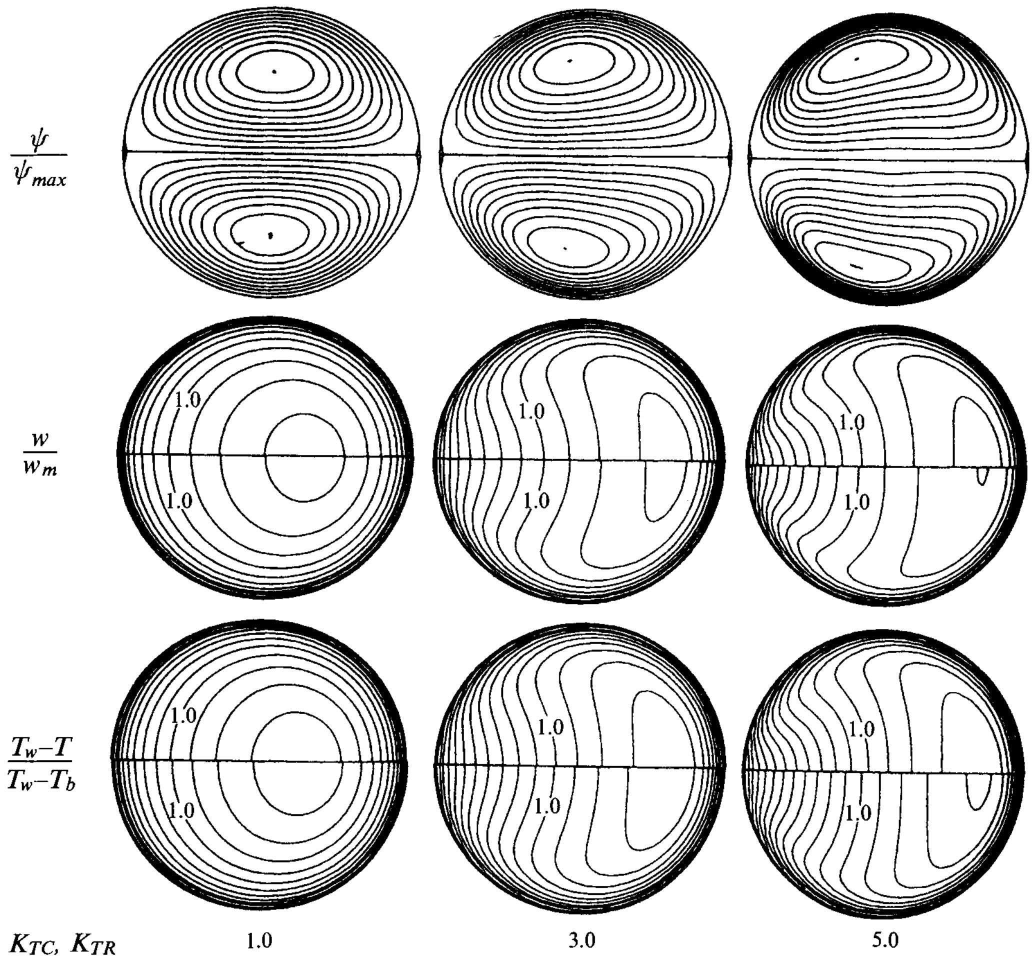

FIGURE 3. Contours of secondary stream function ψ, axial velocity and temperature ($Pr = 0.71$) (upper half: curved pipe flow, lower half: rotating pipe flow).

straight pipe flow. Computational results using a $k-\epsilon$ model are also shown in the figure.

Figure 1 shows that: (i) experimental data for various values of λ or Ro show the similarity of K_{TC} or K_{TR} in each flow, (ii) the two sets of experimental results coincide over the whole range of experiments, (iii) the maximum increase of the friction factor from the Blasius formula is less than 30% at $K_T = 6$, (iv) the computational results for the two flows coincide with each other, but underpredict at larger K_T.

The increase of the friction factors due to the body force is much smaller in turbulent flows than in laminar flows. The increase from turbulent flow without body force may have two causes: the secondary flow and the change of turbulence structure. The secondary flow is a direct effect of body force on the mean flow, since it originates from the body-force terms in the mean flow equations. The change of turbulence structure is an indirect effect, since it originates from the body-force-related terms in the turbulent stress equations, and it affects the mean flow through turbulent stresses. The major cause of the increase in the friction factors seems to be the secondary flows. The effect of a change of turbulent structure due to body force is not considered in the computation. The underprediction in the computation may come from this effect.

As the two sets of experimental results coincide, either can be used to estimate both flows. Ito's formula for curved pipe flows, which has a wider application range, can be rewritten for the Fanning friction factor $f = \bar{\tau}_w/(\frac{1}{2}\rho w_m^2)$, where $\bar{\tau}_w$ is a peripheral average wall friction, as

$$fk_2^{1/2} = 0.005\,13 + 0.0760/K_T, \tag{1}$$

where K_T and K_2 represent K_{TC} and λ for curved pipe flows, and K_{TR} and R_0 for orthogonally rotating pipe flow. The range of application is $0.6 < K_T < 6$ and $K_2 > 8$. This is the analogy formula for both turbulent flows.

3.2. *Heat transfer rate*

It is assumed that the analogy is also valid for heat transfer for fixed values of the Prandtl number *Pr*. We shall discuss the mean Nusselt number $Nu = q_w d/[(T_w - T_b)k]$ with a Prandtl number of 0.71, where q_w is heat flux at the wall, T_w the wall temperature, T_b the fluid bulk temperature and k the thermal conductivity of the fluid. The thermal boundary condition of the pipe is an axially constant wall heat flux with a peripherally constant wall temperature, a typical condition for metal pipes. Figure 2 includes experimental data and computational results for both flows, as well as an empirical formula by Kays & Crawford (1980) for non-rotating straight pipe flow for reference. Experimental data for air are taken from Mori & Nakayama (1967) for curved pipe flow and from Mori *et al.* (1971) for rotating pipe flow. The experimental data of the latter were given in terms of their own complicated composite parameter, without enough information to evaluate *Ro*. Therefore, K_{TR} is estimated by assuming two values of $Ro = 10$ and 100, and these data are shown by two symbols which deviate from the computational curve only slightly. Comparing with the Kays–Crawford formula, the maximum increase of *Nu* due to body force is about 20% at $K_T = 6$.

The computational results for these two flows coincide with each other, and experimental data for the two flows agree quite well with the computations. The computational results for the two flows can be expressed by

$$Nu/K_2^{8/5} = 0.0192 K_T^{3.30}, \tag{2}$$

where K_T and K_2 as in (1). This is the analogy formula for the mean Nusselt number for both turbulent flows when $Pr = 0.71$.

3.3. *Flow patterns*

As it is important to see how the two flow patterns are similar for values of $K_{TC} = K_{TR} = K_T$, computed contours for the non-dimensional secondary streamline, axial velocity and temperature ($Pr = 0.71$) are shown in figure 3 for three values of K_T. The upper half of the pipe cross-section shows curved pipe flow, while the lower half shows rotating pipe flow. For all three values of K_T, the contours of the two flows are very similar, particularly for secondary streamlines. For curved pipe flow, comparisons between contours computed with a standard $k-\epsilon$ model and contours measured by Rowe (1966) were made by Patankar *et al.* (1975), which showed a fairly good agreement (see also Rodi 1984).

4. Conclusion

Following a previous paper which had shown an analogy between laminar flows, the quantitative analogy between fully developed turbulent flows in curved pipes and in

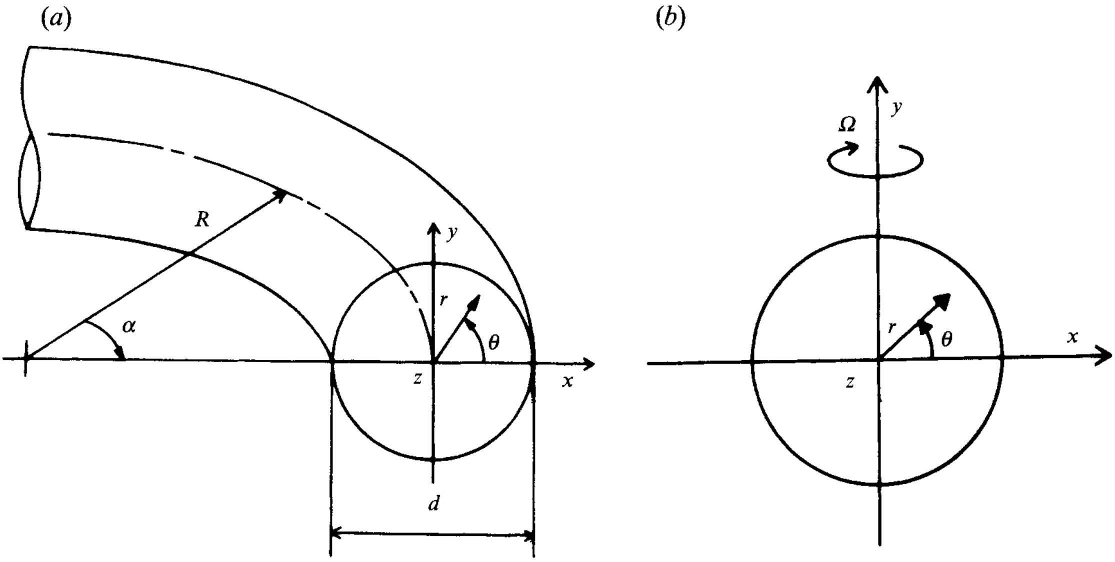

FIGURE 4. Configuration of (*a*) curved pipe flow, (*b*) orthogonally rotating pipe flow.

orthogonally rotating pipes has been demonstrated through similarity arguments, use of experimental data and computational results. Dimensionless governing parameters K_{TC} and K_{TR} having the same dynamical meaning as those for laminar flows were introduced. The second parameters λ and Ro were the same as in laminar flows. For λ and Ro large enough, K_{TC} and K_{TR} became the sole governing parameters in their respective flows, so the analogy between the two turbulent flows became evident. With the new scalings, experimental data showed that the friction factor and the mean Nussel number of the two flows coincided respectively, when $K_{TC} = K_{TR}$. Contours of primary and secondary velocity, as well as of temperature for $Pr = 0.71$, were shown to be similar for a wide range of the parameters.

Appendix. Outline of the computation

In curved pipe flow, it is assumed in the equations that λ is large enough for its effect to be negligible. As shown in figures 4(*a*) and 4(*b*), cylindrical polar coordinates (r, θ, z) fixed to a pipe are taken for both flows with corresponding velocities (V_r, V_θ, V_z). A standard $k-\epsilon$ model of turbulence with a wall function is employed in the computation. In the fully developed region, the equations of continuity, motion, temperature T, turbulent energy k and its dissipation rate ϵ are written as follows.

The equation of continuity is

$$\frac{\partial}{\partial r}(rV_r)+\frac{\partial}{\partial \theta}V_\theta = 0. \tag{A 1}$$

Reynolds equations can be written, after standard approximations, in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\rho V_r \phi - \tilde{\mu}\frac{\partial \phi}{\partial r}\right)\right]+\frac{\partial}{r\partial \theta}\left(\rho V_\theta \phi - \tilde{\mu}\frac{\partial \phi}{r\partial \theta}\right) = P_\phi + F_\phi + S_\phi, \tag{A 2}$$

where ϕ stands for V_r, V_θ, V_z, T, k or ϵ. The diffusion coefficient $\tilde{\mu}$, pressure-gradient

ϕ	$\tilde{\mu}$	P_ϕ Curved flow	P_ϕ Rotating flow	F_ϕ Curved flow	F_ϕ Rotating flow	S_ϕ
V_r	μ_{eff}	$-\dfrac{\partial p}{\partial r}$	$-\dfrac{\partial p^*}{\partial r}$	$\dfrac{\rho V_z^2}{R}\cos\theta$	$2\rho\Omega V_z\cos\theta$	$\dfrac{\rho V_\theta^2}{r}-\mu_{eff}\left(\dfrac{2}{r^2}\dfrac{\partial V_y}{\partial\gamma}+\dfrac{V_r}{r^2}\right)+\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\mu_T\dfrac{\partial V_r}{\partial r}\right)+\dfrac{\partial}{r\partial\theta}\left(\mu_T\dfrac{\partial V_\theta}{\partial r}\right)-\dfrac{\mu_T V_r}{r^2}-\dfrac{\partial}{r\partial\theta}\left(\mu_T\dfrac{V_\theta}{r}\right)$
V_θ	μ_{eff}	$-\dfrac{1}{r}\dfrac{\partial p}{\langle\theta}$	$-\dfrac{1}{\mathrm{r}}\dfrac{\partial p^*}{\partial\theta}$	$-\dfrac{\rho V_z^2}{R}\sin\theta$	$-2\rho\Omega V_z\sin\theta$	$-\dfrac{\rho V_r V_\theta}{r}+\mu_{eff}\left(\dfrac{2}{r^2}\dfrac{\partial V_r}{\partial\theta}-\dfrac{V_\theta}{r^2}\right)+\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\mu_T\dfrac{\partial V_r}{r\partial\theta}\right)+\dfrac{\partial}{r\partial\theta}\left(\mu_T\dfrac{\partial V_\theta}{r\partial\theta}\right)+\dfrac{2V_r}{r^2}\dfrac{\partial\mu_T}{\partial\theta}-\dfrac{V_\theta}{r}\dfrac{\partial\mu_T}{\partial r}+\dfrac{\mu_T}{r^2}\dfrac{\partial V_r}{\partial\theta}$
V_z	μ_{eff}	$-\dfrac{\partial p}{\partial z}$	$-\dfrac{\partial p^*}{\partial z}$	0	$-2\rho\Omega(V_r\cos\theta - V_\theta\sin\theta)$	0
k	$\dfrac{\mu_T}{\sigma_k}$	0	0	0	0	$G-\rho\epsilon$
ϵ	$\dfrac{\mu_T}{\sigma_\epsilon}$	0	0	0	0	$C_1 G\dfrac{\epsilon}{k}-C_2\rho\dfrac{\epsilon^2}{k}$

where $\mu_{eff}=\mu+\mu_T$, $\mu_T=C_\mu\rho k^2/\epsilon$

$$\frac{G}{\mu_T}=2\left(\frac{\partial V_r}{\partial r}\right)^2+2\left(\frac{\partial V_\theta}{r\partial\theta}\right)^2+\left(\frac{\partial V_z}{\partial r}\right)^2+\left(\frac{\partial V_z}{r\partial\theta}\right)^2+\left(\frac{\partial V_r}{r\partial\theta}+\frac{\partial V_\theta}{\partial r}\right)^2 -\frac{2V_\theta}{r}\left(\frac{\partial V_r}{r\partial\theta}+\frac{\partial V_\theta}{\partial r}\right)+\frac{2V_r}{r}\left(\frac{V_r}{r}+\frac{2}{r}\frac{\partial V_\theta}{\partial\theta}\right)+\left(\frac{V_\theta}{r}\right)^2,$$

$C_\mu=0.09$, $C_1=1.44$, $C_2=1.92$, $\sigma_k=1.0$, $\sigma_\epsilon=1.3$.

TABLE 1. The coefficients and terms for the variable ϕ

term P_ϕ, body-force term F_ϕ and source term S_ϕ for both flows are given in table 1. The p^* in rotating pipe flow is the reduced pressure given by

$$p^* = p - \tfrac{1}{2}\rho\Omega^2(r^2\cos^2\theta + z^2). \tag{A 3}$$

The assumption that Ro is large enough means that the F_ϕ term in the V_z equation for rotating pipe flow is small enough. The model constants in the k- and ϵ-equations have standard values, as shown in table 1. The wall function to give boundary conditions at near-wall grids is described in Launder & Spalding (1974), and a detailed description of the method is given in Patankar *et al.* (1975).

The numerical scheme employed to solve these equations is based on the finite-volume approach, which is an adaptation of that of Patankar (1980). The computational grid covers only a semicircular sector because of the symmetry of the flow with respect to the x-axis. The grid density employed is 30 in both r- and θ-directions. In the r-direction, the location of the near-wall grids are specified so as to satisfy the condition $30 < y^+ \ (= y_w u_\tau/\nu) < 100$, where y_w is the distance from the wall and $u_\tau = (\tau_w/\rho)^{1/2}$ is the friction velocity.

The convergence criterion is specified with all the normalized residual errors for dependent variables to be less than 10^{-6}. Computations for various values of K_{TC} or K_{TR} are made by fixing Re at 10^5 and changing λ or Ro while keeping λ, $Ro > 8$.

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